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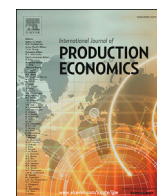
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The effects of contractual agreements on the economic production quantity model with machine breakdown

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ABSTRACT

This paper develops a production-inventory model which is subject to breakdowns, and studies the influence of outsourcing on the expected total cost and the fill rate in case of any failure in the production facility. To avoid shortages and aim at a higher fill rate when there are random breakdowns, the manufacturer has the option to purchase some quantities from an external supplier while repairing the production facility. In this paper, this transaction is formulated through different settings. First, the manufacturer has the option to purchase the items from an available supplier in the market. The manufacturer is also given the option to procure the required items from a predetermined supplier based on a contractual agreement. These scenarios are then compared with the setting in which the manufacturer keeps safety stock in case of breakdown. The results of this study show that using an external supplier, when the machine is prone to failures, improves the performance of the system. We have also shown that it is more beneficial for the manufacturer to collaborate with an external supplier rather than keep safety stock. The analysis is further elaborated using several numerical experiments.

1. Introduction

In a competitive market where end-users are offered a large variety of substitute items, an out of stock situation may swiftly result in losing the margins that a business could have gained had it met the demand. The costs of shortages may extend to negatively affecting the future demand of the firm. The importance of goodwill, in a highly competitive market, forces firms to adopt strategies which will not only meet the demand they receive but also increase their market share. To avoid or minimize unplanned stock-outs in an inventory system, when defining the economic order quantity (EOQ), companies normally take proactive measures, such as diversifying the supplier portfolio or considering safety stock, as a hedge against supply disruptions.

In the production sector, factors such as unpredictable breakdowns in the production machinery, failure in transportation, and low-quality raw material may disrupt the planned supply flow. What if a manufacturer encounters a breakdown in the manufacturing process? How long would

the customers wait for their items? What if these breakdowns are unavoidable due to the characteristics of the process? In a production-inventory system when a manufacturer sets their economic production quantity (EPQ), they should always take into account the reliability of the system. Since breakdowns are inevitable, a manufacturing firm should always have a contingency plan to deal with such situations. This issue has been addressed by the research community in recent decades. In addition to corrective maintenance, it has been suggested that a preventive maintenance plan, safety stock and inspection/rework operation, either individually or in combination, could be used to mitigate the effects of a disrupted production process.

The term “emergency replenishment” has been used in the inventory management literature to address an inventory system with constant supply flow (without disruptions) and stochastic demand (see, e.g., Axsäter, 2014; Johansen and Thorstenson, 2014). In such models, when there is a leap in the demand rate, researchers suggest the use of an emergency order that has a shorter lead time but incurs higher costs

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compared with ordinary orders. In this paper, we consider such a replenishment as an option for the manufacturer who replenishes his/her stock by ordering from an available supplier. We also examine other scenarios in which the manufacturer purchases some quantities from a specific supplier by long-term agreement. For example, items such as printed boxes, plastic bottles or computer chips could be produced by other manufacturers after they had made minor adjustments to their production facilities. Although buying items from an external supplier would result in some loss in the margins, it could compensate for part of the overhead cost, protect the manufacturer's reputation, and ensure future demand. The findings of this paper confirm that using an external supplier, by a contractual agreement, to meet the demand while the machine is being repaired, is beneficial.

The contribution of this research work is twofold. First, we introduce a new alternative solution to be considered when breakdowns are common in a business. The solution is more suitable for those production environments in which machine failure is frequent (e.g. relatively old machinery but not too old to be replaced) and when it is less economical in the long term to receive emergency replenishments from available suppliers. Second, the paper provides a tool to analyze and formulate the collaboration settings ensuring that both parties benefit from the joint venture.

The remainder of the paper is structured as follows: Section 2 presents an overview of the relevant literature; Section 3 describes the system as well as the mathematical model; and Section 4 presents the mathematical analysis of the system under different scenarios. Section 5 summarizes the safety stock policy introduced in the literature, followed by numerical analyses in Section 6. Finally, conclusions and insights are summarized in Section 7.

2. Literature review

Machine failure is a widespread incident in a manufacturing environment, and this has motivated the research community to address such problems. Researchers have modeled machine unreliability and its outcomes in different ways. In this section, we give an overview of the literature which focuses on these problems and on the model introduced in this paper.

One of the first papers to conceptualize random breakdowns was published by Kimemia and Gershwin (1981) in which they introduced a flexible manufacturing system (FMS) designed to produce a group of items with similar production requirements. The FMS consists of a set of machinery with an adjustment to the production plan which allows the rest of the machines to continue production while a faulty machine is undergoing repairs. Akella and Kumar (1986) introduced a similar model in which the production rate is optimized as a function of on-hand inventory. Bielecki and Kumar (1988) analyzed a particular case of Kimemia and Gershwin (1981) and showed that zero-inventory policies resulted in optimality for the systems with uncertainties.

Posner and Berg (1989) considered a machine with a constant production rate that produces an item for which the demand follows a Poisson distribution. They showed that operation time before failure and repair time both follow an exponential distribution. Groenevelt et al. (1992b) studied two different production policies under the condition of an unreliable machine. The researchers assumed that after the machine was fixed the firm could either continue the previous lot or start a new one. Berg et al. (1994) analyzed a system with multiple machines whose operating time before failure had an exponential distribution. The demand rate was considered to be based on a Poisson distribution, and any unmet demand lost. They obtained the distribution of the inventory level at the production-inventory system and used that to evaluate the performance of the process.

Glock (2013) considered the assumptions made in the literature for unreliable manufacturing systems and demonstrated that some models needed adjustments to avoid irrational results, e.g., a decrease in the total cost caused by breakdowns. He divided papers on machine breakdown

into three different categories, based on what happens to the functionality of the machine and on the quality of the item produced: 1) the machine remains in operation, however, a proportion of the items produced are defective (see Section 2.3); 2) the condition of the machine lies between fully functional and not functional, hence its output is reduced; and 3) the machine is not functional. Although our study falls into Glock's third category, we have used a different system of categorization because we are interested in what measures should be taken when a production system is unreliable.

2.1. Preventive maintenance

There is a vast literature on preventive maintenance plans for the production sector in which researchers combine lot-sizing decisions with maintenance planning. In such systems, the production facility is (mostly) shut down to perform the planned preventive maintenance. In an early study, Kamien and Schwartz (1971) developed an optimal maintenance plan for a machine and suggested an appropriate time-to-sell for the machine before it started to fail frequently.

Groenevelt et al. (1992a) suggested that a fraction of the production should be stored as safety stock when the machinery was operating. The authors, simultaneously found the optimal plan for the preventive maintenance operation. Cheung and Hausman (1997) optimized preventive maintenance and safety stock with a general time-to-failure distribution function, assuming that there is a possibility of machine breakdown. By relaxing some assumptions, Dohi et al. (2001) analyzed the model introduced by Cheung and Hausman (1997) with a more precise expected cost function. Giri et al. (2005) considered production rate as a decision variable to optimize the system while time-to-breakdown and repair time are stochastic. It was also assumed that the failure rate of the machine was linked to the production rate since the stress level on the machine changes with the production rate. Aghezzaf et al. (2007) analyzed a multiple-item production system subject to random breakdowns that aimed to meet the demands of all items over a finite planning horizon and eventually suggested a joint production and maintenance planning model. El-Ferik (2008) examined a production system with age-based maintenance policy and an increasing failure rate. The researcher suggested a preventive maintenance cycle, and if the machine stops before these time points, the maintenance operations immediately start. Kazaz and Sloan (2013) assumed that a process deteriorates over time, its functionality existing somewhere on the spectrum between its best and worst state. Moreover, this process produces multiple items that can be treated with different maintenance plans. Zhang et al. (2014) used a dynamic method for estimating the size of production lots which took machine failures into account, and minimized the average cost instead of the expected cost. Paul et al. (2015) considered a single stage production-inventory system with random disruption. Their model maximizes the total profit during the recovery time window by generating a revised maintenance plan after the breakdown occurs.

2.2. Safety stock

The accumulation of safety stock is one of the measures taken by businesses to avoid shortages in the event of a mismatch between supply and demand. The literature on unreliable manufacturing processes indicates that researchers analyze the performance of a system by assuming that both safety stock and a preventive maintenance plan are in place (see, e.g., Groenevelt et al., 1992a; Cheung and Hausman, 1997; Dohi et al., 2001; Giri et al., 2005 in Section 2.1). Sana and Chaudhuri (2010) designed a production-inventory model that conducted preventive maintenance and kept safety stock. The authors checked the status of the process based on the quality of the last item of the batch, and if it was of acceptable quality, then the whole batch was accepted. In this system, the preventive maintenance operations were performed only if there was no breakdown and a certain level of inventory had accumulated. They then

numerically analyzed the model to find the optimal policy. Chiu et al. (2011) developed a model which considered the safety stock level at a very early stage of production as a means of guarding against a machine breaking down and continuing to produce defective items none of which can be used. Bouslah et al. (2013) optimized both production rate and lot-sizing in an unreliable system, and used an acceptance sampling technique to detect and discard the defective items.

2.3. Imperfect quality, inspection, and rework

Unreliable production processes that generally produce items of acceptable quality for a period (“in-control” state) before producing defective items (“out-of-control” state) are discussed in the academic literature (see, e.g. Rosenblatt and Lee, 1986). Researchers usually suggest that an inspection is performed on the items, which incurs a fixed cost per lot and per item inspected. Boone et al. (2000) studied a production system which when in an out-of-control state continues to produce defective items. The study assumed that the system incurs a fixed cost for each unit of defective item produced.

Chakraborty et al. (2008) introduce a generalized variant of the model developed by Boone et al. (2000) with stochastic time-to-breakdown, repair time, and maintenance time. In their research, Chakraborty et al. (2008) assumed that preventive maintenance was conducted at the end of each production period. Chakraborty et al. (2009) developed a similar production system but with different inspection policies: if the machine was found to be out-of-control then corrective maintenance was undertaken; and if found to be in-control, either no action was taken (policy I) or some preventive maintenance was performed (policy II). In the studies done by Chakraborty et al. (2008, 2009) defective items linearly contributed to the expected total cost of the system; however, it was unclear as to the source of this cost. Depending on the complexity of the item, this cost could be in the form of reworking cost, disposal cost, or inspection cost.

Liao et al. (2009) optimized both the maintenance and production plans. They assumed that preventive maintenance may be insufficient to fully restore the production process and that the defective items could be reworked to reach an acceptable level of quality. Sana (2010) developed a model in which the production rate (a decision variable) varied during the production period: when in an out-of-control state, a different percentage of the items produced are defective and need reworking. Sarkar and Saren (2016) considered a situation in which the inspection operation is subject to error. In the event that defective items do reach the customer, the manufacturer offers a warranty to the customer.

In this paper, we analyze a production-inventory system which is susceptible to failures. We assume that when a breakdown occurs, the manufacturer has the option to fulfill the demand using an external supplier. We formulate this collaboration through three different scenarios and further conduct analyses to compare them: (1) the manufacturer approaches a supplier in the spot market with no contractual commitment for an emergency replenishment, (2) the manufacturer purchases items based on a revenue-sharing contract, and (3) the manufacturer has a contractual agreement with price discount. Moreover, we compare these scenarios with the case when the manufacturer decides to keep some levels of safety stock.

3. Model description

3.1. Notations

The following notations are used throughout the paper:

- Q , production lot size (units);
- Q' , manufacturer's order quantity to external supplier (units);
- P , production rate (units/unit of time);
- D , demand rate (units/unit of time);
- A , fixed ordering cost for raw material (\$/order);

- A' , fixed ordering cost for purchased items from the supplier (\$/order);
- S_0 , probability of having a supplier available in the market;
- C , unit production cost at the manufacturer (\$/unit);
- C' , unit sales price of the finished good at the supplier (\$/unit);
- C_r , machine repair cost (\$/unit of time);
- C_p , sales price of the item at the manufacturer (\$/unit);
- λ , machine failure rate (failures/unit of time);
- μ , machine repair rate (failures/unit of time);
- π , unit lost sale cost (\$/unit);
- f , out-of-pocket holding cost at the manufacturer (\$/unit/unit of time);
- i , capital cost rate (1/unit of time);
- h , holding cost rate for produced items (\$/unit/unit of time);
- h' , holding cost rate for purchased items (\$/unit/unit of time);
- T , cycle time (unit of time).

3.2. The model

In this production-inventory model, a manufacturer produces an item at the rate of P to meet a constant demand rate of D . The initiation of the production process incurs a fixed cost of A and a variable production cost of C per item. The inventory holding cost is $h = iC + f$ per unit of item per unit of time, where i is the capital rate and f is out-of-pocket unit holding cost. The production time before breakdown (t_w) and repair time (t_r), both follow an exponential distribution. The aim is to maximize the expected total profit or (if equivalent, see, e.g. Ghiami and Beullens, 2016) to minimize the expected total cost.

Under perfect conditions where there is no machine failure, the manufacturer would plan for the optimal lot size derived from EPQ that suggests setting inventory cycle and production period at $t^* = \frac{Q}{D} = \sqrt{\frac{2A}{hD}} \sqrt{\frac{P}{P-D}}$ and $t_p^* = \frac{Q}{P} = \sqrt{\frac{2DA}{h}} \sqrt{\frac{1}{P(P-D)}}$, respectively. In the current system, however, the manufacturer may encounter machine failures. To be prepared for such situations, we assume that the manufacturer has an external source for replenishment. After the production process starts, the manufacturer may face five different situations during one inventory cycle, each of which takes place with a specific probability. The manufacturer starts the production period and with a probability of $Pr(t_p^* < t_w)$ continues producing for t_p^* units of time with no breakdown (Case 1). If the machine fails before the production runtime of t_p^* , then the machine undergoes the repair operation. Case 2 occurs with a probability of $Pr(t_r < t_o, t_w < t_p^*)$ which is when the repair operation comes to an end before the on-hand inventory is completely depleted (t_o). The manufacturer may not be able to repair the machine before running out of inventory. Therefore, he or she may replenish it with the aid of an external source that has a reliability of S_0 . Case 3 occurs with probability $(1 - S_0) Pr(t_o < t_r, t_w < t_p^*)$ when the external supplier entirely fails to supply the items. In case 4, there is a chance that the manufacturer receives a sufficient quantity from the supplier to cover demand for a period of t_m units of time. The manufacturer, then, uses that quantity while the repair operation proceeds. With a probability of $S_0 Pr(t_o < t_r < t_o + t_m, t_w < t_p^*)$ the repair operation finishes before the manufacturer runs out of inventory (Case 4). There is a chance (Case 5) that the manufacturer starts a stock-out period (after finishing the received quantity from the supplier) before the machine is fully functional. This occurs with a probability of $S_0 Pr(t_o + t_m < t_r, t_w < t_p^*)$. In Sections 3.2.1–3.2.5, we present mathematical analyses for these cases.

3.2.1. Case 1

The desired situation would be to have no disruptions in the production process and hence set inventory cycle and production period at $t^* = \frac{Q}{D}$ and $t_p^* = \frac{Q}{P}$, respectively. Since the production process is not fully reliable, the firm may face breakdowns. For each production cycle, the

machine works for a period of t_w before it breaks, or, the manufacturer may face no breakdowns with the probability of $p_1 = \Pr(t_p^* < t_w)$. This probability could be obtained as

$$p_1 = \int_{t_p^*}^{\infty} \Pr(t_w) dt_w = e^{-\lambda t_p^*}. \quad (1)$$

Fig. 1 depicts the inventory pattern in Case 1. In this case, the inventory period and the total cost of the system during this period are

$$t_1 = \frac{Pt_p^*}{D} = \frac{Q}{D}, \quad (2)$$

and

$$TC_1 = A + \frac{h}{2} \frac{(P-D)D}{P} t_p^{*2} + CP_{t_p^*}, \quad (3)$$

respectively.

3.2.2. Case 2

In the event that machine failure occurs before manufacturing the planned lot size, ($t_w < t_p^*$), the manufacturer is left with an on-hand inventory of $Q_w = (P-D)t_w$ at the time of the breakdown. The manufacturer must simultaneously repair the machine and meet the demand using Q_w . The repair operation takes t_r units of time and is a random variable with exponential distribution. It takes $t_o = \frac{(P-D)t_w}{D}$ time units for the manufacturer to use up all of the stock accumulated prior to the breakdown. Should the machine return to operation before Q_w depletes ($t_r < t_o$), the manufacturer restarts the production process once all the on-hand stock has been exhausted (see Fig. 2). The probability of facing Case 2 is $p_2 = \Pr(t_r < t_o, t_w < t_p^*)$, therefore

$$p_2 = \int_0^{t_p^*} \lambda e^{-\lambda t_w} \int_0^{t_o} \mu e^{-\mu t_r} dt_r dt_w = (1 - e^{-\lambda t_p^*})(1 - e^{-\mu t_o}). \quad (4)$$

The inventory period and the total cost of the system are hence given by

$$t_2 = \frac{Pt_w}{D}, \quad (5)$$

and

$$TC_2 = A + \frac{h}{2} \frac{(P-D)P}{D} t_w^2 + CP_{t_w} + C_r t_r, \quad (6)$$

respectively.

3.2.3. Case 3

The repair operation may take longer than the period for which Q_w

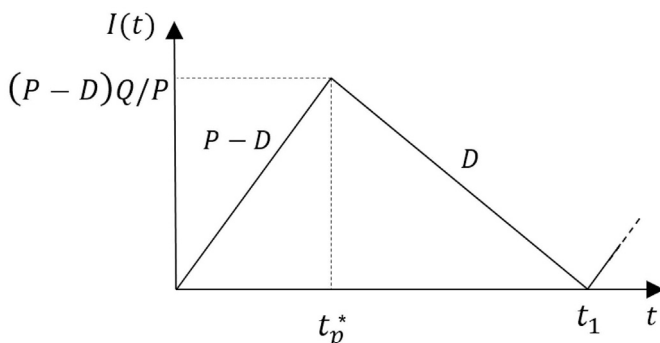


Fig. 1. Manufacturer's inventory level if Case 1 happens.

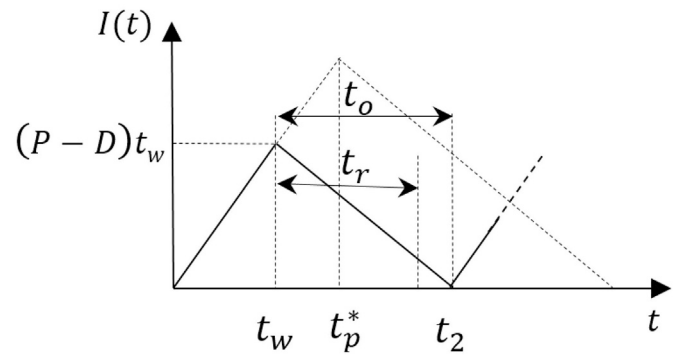


Fig. 2. Manufacturer's inventory level if Case 2 occurs.

can cover the demand ($t_r > t_o$). In such a situation, the manufacturer buys a quantity of Q' from the market. There is, however, an overall service level for the market, and the manufacturer can obtain the desired items with probability S_θ . Should the market be unable to provide the manufacturer with the necessary items ($1 - S_\theta$), the system will face a shortage and lose demand until the machine is repaired (see Fig. 3). The probability of this occurring is therefore $p_3 = (1 - S_\theta)\Pr(t_o < t_r, t_w < t_p^*)$:

$$p_3 = (1 - S_\theta) \int_0^{t_p^*} \lambda e^{-\lambda t_w} \int_{t_o}^{\infty} \mu e^{-\mu t_r} dt_r dt_w = (1 - S_\theta) e^{-\lambda t_p^*} (1 - e^{-\mu t_o}). \quad (7)$$

In this case, the inventory cycle is completed when the machine is repaired and starts a new production batch, therefore:

$$t_3 = t_w + t_r. \quad (8)$$

The total cost of the system is then given by

$$TC_3 = A + \frac{h}{2} \frac{(P-D)P}{D} t_w^2 + CP_{t_w} + C_r t_r + \pi(Dt_r - (P-D)t_w). \quad (9)$$

3.2.4. Case 4

In the event that the supplier can meet the manufacturer's demand of Q' , the manufacturer incurs a fixed ordering cost of A' when purchasing the quantity. The purchasing price for this quantity is C' per item. The inventory holding cost for these items, hence, incurs at the rate of $h' = iC' + f$. The manufacturer meets the demand using the new lot received from the market. With this new lot, it takes $t_m = Q'/D$ units of time before the inventory level reaches zero, as Fig. 4 shows. This situation occurs with probability $p_4 = S_\theta \Pr(t_o < t_r < t_o + t_m, t_w < t_p^*)$.

The inventory cycle in this case is equal to

$$t_4 = \frac{Pt_w}{D} + t_m, \quad (10)$$

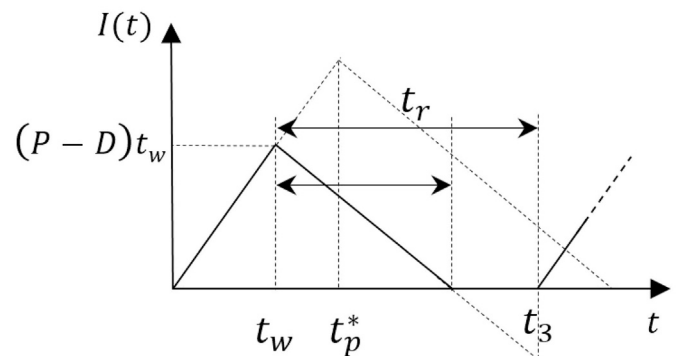


Fig. 3. Manufacturer's inventory level in Case 3.

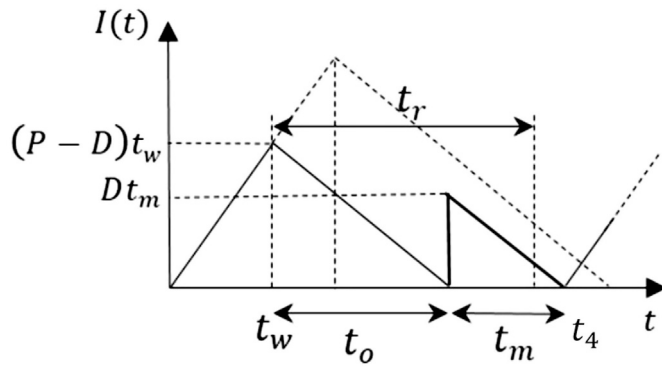


Fig. 4. Manufacturer's inventory level in Case 4.

and the total cost of the system is given by

$$TC_4 = A + \frac{h}{2} \frac{(P-D)P}{D} t_w^2 + CPt_w + C_r t_r + A' + \frac{h'}{2} Dt_m^2 + C' Dt_m. \quad (11)$$

3.2.5. Case 5

The manufacturer purchases a batch of Q' from a supplier and starts using the items to meet the demand while the repair process continues. The probability that the manufacturer runs out of items before the machine is repaired is, $p_5 = S_\theta \Pr(t_o + t_m < t_r, t_w < t_p^*)$. Fig. 5 illustrates the inventory level in this case.

The corresponding inventory cycle for this case is

$$t_5 = t_w + t_r, \quad (12)$$

and the total cost of the system is given by

$$TC_5 = A + \frac{h}{2} \frac{(P-D)P}{D} t_w^2 + CPt_w + C_r t_r + A' + \frac{h'}{2} Dt_m^2 + C' Dt_m + \pi(D(t_r - t_m) - (P-D)t_w). \quad (13)$$

4. Purchasing scenarios

In this paper, we define and analyze three scenarios for the manufacturer to procure the items needed from an external supplier. In Scenario 1, if breakdown occurs, the manufacturer chooses to buy some quantity of goods from a supplier with no previous agreement (emergency replenishment). In the long-run, however, frequent emergency replenishments may be costly and may push the manufacturer towards accepting more shortages rather than placing emergency orders. The primary outcome of this behavior would be a decrease in the service level, and hence some loss in sales. The two parties could circumvent such situations and make some trade-offs (lower margins and higher sales) to reach an agreement that boosts the service level offered by the

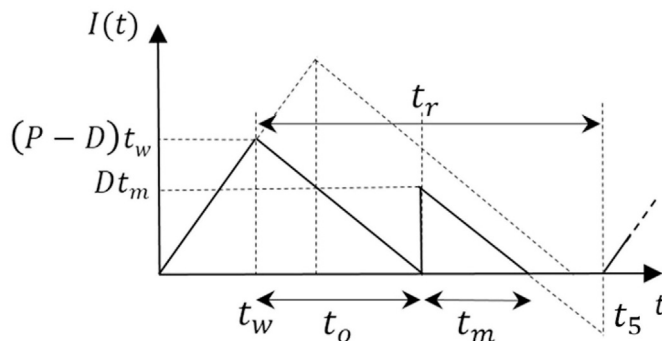


Fig. 5. Manufacturer's inventory level in Case 5.

“integrated supply chain”. It is challenging to quantify such trade-offs and requires consideration of all the main parameters and factors that play a role in the supply chain cost/profit function. The supply chain literature introduces quite a few coordination settings for such contracts (see, e.g. Arani et al., 2016; Hu et al., 2016; Cai et al., 2017; Chen et al., 2017; Guo et al., 2017; Meng et al., 2017; Xiao et al., 2017; Zheng et al., 2017; Song and Gao, 2018), and the two most relevant to this paper are revenue sharing and quantity discount. In Scenario 2 we assume that there is an agreement between the manufacturer and a supplier, based on a revenue-sharing contract. Through this agreement, the manufacturer shares the revenues obtained from the purchased items with the supplier. Finally, we consider Scenario 3 in which the supplier gives the manufacturer a discount. To analyze these scenarios, we calculate and obtain the expected cycle time and expected total cost for each period.

4.1. Scenario 1: Emergency replenishment without contract

In this scenario, the manufacturer may decide not to have a long-term agreement to purchase some quantities when needed. This means, if the stock level drops to zero and the machine is still under repair, the manufacturer can approach a supplier in the market who can supply a quantity of Q' with the probability of S_θ . Considering the cases defined in Section 3.2 the expected inventory cycle is

$$E(T) = \frac{P}{\lambda D} \left(1 - e^{-\lambda \frac{Q'}{P}}\right) + \left(\frac{\lambda}{\mu} (1 - S_\theta) + S_\theta \lambda \frac{Q'}{D} + \frac{S_\theta \lambda e^{-\mu \frac{Q'}{D}}}{\mu} \right) \left(\frac{1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D}\right) \frac{Q'}{P}}}{\lambda + \frac{\mu(P-D)}{D}} \right). \quad (14)$$

Taking into account the probability of the cases and the total cost incurred in each case, the expected total cost of the system is given by

$$E(TC) = A + \left(\frac{\lambda \left(1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D}\right) \frac{Q'}{P}}\right)}{\left(\lambda + \frac{\mu(P-D)}{D}\right)} \right) \left(S_\theta \left(A' + \frac{h' Q'^2}{2} + C' Q' \right) + \frac{\pi D}{\mu} \left(1 - S_\theta + S_\theta e^{-\mu \frac{Q'}{D}} \right) + \left(\frac{C_r}{\mu} + \frac{CP}{\lambda} \right) \left(1 - e^{-\lambda \frac{Q'}{P}} \right) + \frac{h(P-D)}{D} \left(\frac{P \left(1 - e^{-\lambda \frac{Q'}{P}} \right)}{\lambda^2} - \frac{Q e^{-\lambda \frac{Q'}{P}}}{\lambda} \right) \right). \quad (15)$$

For a detailed analysis of the derivations of Equations (14) and (15) see Appendix A.

To evaluate the performance of the manufacturer in meeting the demand, we define the manufacturer's fill rate as:

$$FR = \frac{E(\text{demand met during } T)}{E(\text{demand during } T)} = \frac{\frac{P}{\lambda} \left(1 - e^{-\lambda \frac{Q'}{P}} \right) + S_\theta Q' \lambda \left(\frac{1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D}\right) \frac{Q'}{P}}}{\lambda + \frac{\mu(P-D)}{D}} \right)}{D \left(\frac{P}{\lambda D} \left(1 - e^{-\lambda \frac{Q'}{P}} \right) + \left((1 - S_\theta) \frac{\lambda}{\mu} + \frac{S_\theta Q' \lambda}{D} + \frac{S_\theta \lambda e^{-\mu \frac{Q'}{D}}}{\mu} \right) \left(\frac{1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D}\right) \frac{Q'}{P}}}{\lambda + \frac{\mu(P-D)}{D}} \right) \right)}. \quad (16)$$

4.2. Scenario 2: Contractual agreement with revenue sharing

We consider a scenario in which the manufacturer has an agreement to buy the required items from a specific supplier. According to this agreement, the supplier offers α percent discount on every item that the manufacturer purchases, and in return, the manufacturer offers α' percent of the revenue gained from the items. This means that the manufacturer loses $\alpha' C_p$ units of money on each item purchased from the

supplier while earning $\alpha C'$ due to the discount. Within this new setting, the expected value of the inventory cycle and the fill rate parametrically remains the same as equations presented in (14) and (16), respectively. In the case of breakdowns, however, the expected value of the total cost function changes since the manufacturer accepts some loss in the margins to avoid loss of goodwill. This will change the expected total cost presented in (15) to the following:

$$E(TC^{RS}) = A + Z \left(\frac{\lambda \left(1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D} \right) \frac{Q}{P}} \right)}{\left(\lambda + \frac{\mu(P-D)}{D} \right)} \right) + \left(\frac{C_r}{\mu} + \frac{CP}{\lambda} \right) \left(1 - e^{-\frac{\lambda Q}{P}} \right) + \frac{h(P-D)}{D} \left(\frac{P \left(1 - e^{-\frac{\lambda Q}{P}} \right)}{\lambda^2} - \frac{Q e^{-\frac{\lambda Q}{P}}}{\lambda} \right) + S_\theta Q' (\alpha' C_p - (1 - \alpha) C'), \quad (17)$$

$$\text{where } Z = \left(S_\theta \left(A' + \frac{(iC'(1-\alpha)+f)Q'^2}{2} + C'(1-\alpha)Q' \right) + \frac{\pi D}{\mu} \left(1 - S_\theta + S_\theta e^{-\frac{\lambda Q'}{P}} \right) \right).$$

4.3. Scenario 3: Contractual agreement with price discount

In this scenario, the supplier's sales price (C') depends on the size of the orders placed by the manufacturer, i. e. if the order quantity (Q') is equal to or greater than a predefined level, B , the supplier would offer β percent discount on the sales price, otherwise the manufacturer should pay the full price for the items. This discounted price is defined as follows:

$$C'_{\text{discount}} = \left((1 - \beta) \min \left\{ \frac{B}{Q'}, 1 \right\} + \beta \right) C'. \quad (18)$$

In this scenario $E(T)$ and FR parametrically remain unchanged as in (14) and (16), respectively. The expected total cost function would stay the same as (15) if the manufacturer orders a quantity less than B , otherwise, the expected total cost is obtained as

$$E(TC^{PD}) = A + Z \left(\frac{\lambda \left(1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D} \right) \frac{Q}{P}} \right)}{\left(\lambda + \frac{\mu(P-D)}{D} \right)} \right) + \left(\frac{C_r}{\mu} + \frac{CP}{\lambda} \right) \left(1 - e^{-\frac{\lambda Q}{P}} \right) + \frac{h(P-D)}{D} \left(\frac{P \left(1 - e^{-\frac{\lambda Q}{P}} \right)}{\lambda^2} - \frac{Q e^{-\frac{\lambda Q}{P}}}{\lambda} \right), \quad (19)$$

$$\text{where } Z = \left(S_\theta \left(A' + \frac{(iC'_{\text{discount}}+f)Q'^2}{2} + C'_{\text{discount}}Q' \right) + \frac{\pi D}{\mu} \left(1 - S_\theta + S_\theta e^{-\frac{\lambda Q'}{P}} \right) \right).$$

5. Safety stock policy

This section analyses the policy of keeping safety stock. The results of this policy are used as a reference to evaluate the performance of the purchasing scenarios. We assume that the manufacturer keeps S_s units of the item as safety inventory. This inventory is used when the repair operation takes a long time and the manufacturer consumes all the on-hand inventory to fulfill the demand. If the safety stock is completely depleted, then there will be a stock-out period until the machine is repaired and becomes fully functional. Some researchers have already explored this policy, and we adopt the model developed by Giri et al. (2005). To make the model comparable with the model developed in our

research, we relax some minor assumptions (e. g. the unit production cost is a function of the production rate, whereas in our paper it is a constant) of the model introduced by Giri et al. (2005). We then calculate the expected cycle time, the expected total cost, and the expected fill rate as follows

$$E(T^{S_s}) = \frac{P}{\lambda D} \left(1 - e^{-\frac{\lambda Q}{P}} \right) + \frac{\lambda \left(P - D e^{-\frac{\lambda Q}{P}} \right)}{\mu(P-D)} \left(\frac{1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D} \right) \frac{Q}{P}}}{\lambda + \frac{\mu(P-D)}{D}} \right), \quad (20)$$

$$E(TC^{S_s}) = A + \left(\frac{C_r}{\mu} + \frac{CP}{\lambda} \right) \left(1 - e^{-\frac{\lambda Q}{P}} \right) + \frac{\lambda \left(1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D} \right) \frac{Q}{P}} \right)}{\lambda + \frac{\mu(P-D)}{D}} \left(\frac{CPD \left(1 - e^{-\frac{\lambda Q}{P}} \right)}{\mu(P-D)} + \frac{\pi D e^{-\frac{\lambda Q}{P}}}{\mu} \right) + \frac{hP}{\mu(P-D)} \left(S_s - D + \frac{D e^{-\frac{\lambda Q}{P}}}{\mu} \right) + \frac{h}{D\lambda} \left(P \left(S_s + \frac{P-D}{\lambda} \right) \left(1 - e^{-\frac{\lambda Q}{P}} \right) - (P-D) Q e^{-\frac{\lambda Q}{P}} \right), \quad (21)$$

and

$$FR^{S_s} = \frac{\frac{P}{\lambda} \left(1 - e^{-\frac{\lambda Q}{P}} \right) + \left(\frac{D\lambda \left(1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D} \right) \frac{Q}{P}} \right)}{\mu(P-D) \left(\lambda + \frac{\mu(P-D)}{D} \right)} \right) \left(P - P e^{-\frac{\lambda Q}{P}} \right)}{\frac{P}{\lambda} \left(1 - e^{-\frac{\lambda Q}{P}} \right) + \left(\frac{D\lambda \left(1 - e^{-\left(\lambda + \frac{\mu(P-D)}{D} \right) \frac{Q}{P}} \right)}{\mu(P-D) \left(\lambda + \frac{\mu(P-D)}{D} \right)} \right) \left(P - D e^{-\frac{\lambda Q}{P}} \right)}, \quad (22)$$

respectively.

6. Numerical experiment

In this production-inventory model, after obtaining the expected total cost of the system and also the expected inventory cycle, we use the renewal reward theorem (see Ross, 2014) to optimize the system:

$$\text{Min } f(Q, Q') = \lim_{t \rightarrow \infty} E \left(\frac{TC}{T} \right) = \frac{E(TC)}{E(T)} \quad (23)$$

Given that we have two decision variables in all of the scenarios introduced, as well as in the safety stock policy, we have used an exhaustive search algorithm to enumerate them.

In order to investigate the performance of this system under different contractual agreements, we consider a dataset in which $P = 800$, $D = 700$, $i = 0.2$, $f = 0$, $C = 20$, $C' = 25$ (therefore $h = 4$, $h' = 5$), $A = 100$, $A' = 120$, $S_\theta = 0.9$, $C_r = 80$, $\lambda = 0.6$, $\mu = 0.8$, $\pi = 40$, $C_p = 60$. After conducting a thorough analysis using different parameters, we report on the most significant findings.

6.1. Revenue sharing contract

Part of our analysis entails studying the effects of α and α' on the optimal policies. We consider values between 0 and 1 for these two parameters and present the results in Fig. 6. The figure shows that if the supplier does not offer any discount ($\alpha = 0$), the manufacturer will not share revenues, $\alpha' = 0$. This situation represents Scenario 1 where there is no collaboration between the two players and the manufacturer operates the system with an expected total cost of 18,000 and a fill rate of 85%. It is worth noting that optimizing this production system without considering an external supplier results in an expected total cost of 19,824 with a fill rate of 61%. This finding shows that giving consideration to an external supplier in the models presented in the literature may bring about an improvement in the performance of those systems.

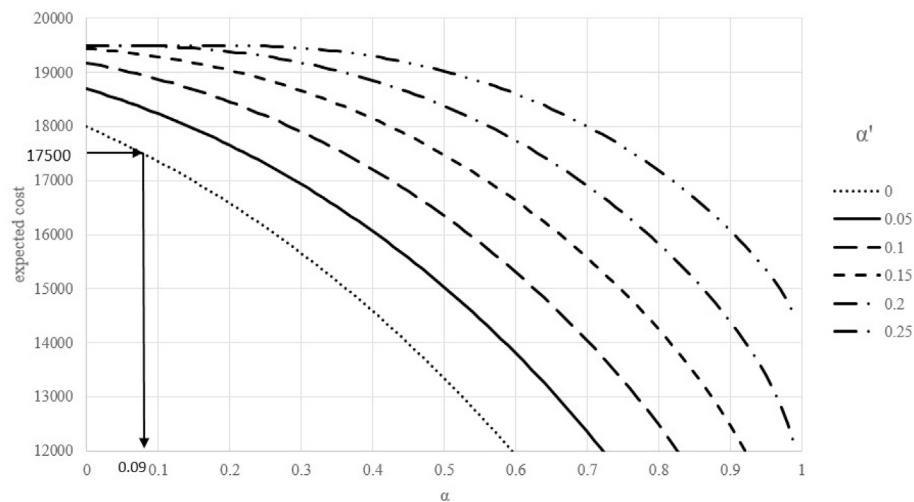


Fig. 6. Expected total cost for different values of α and α' .

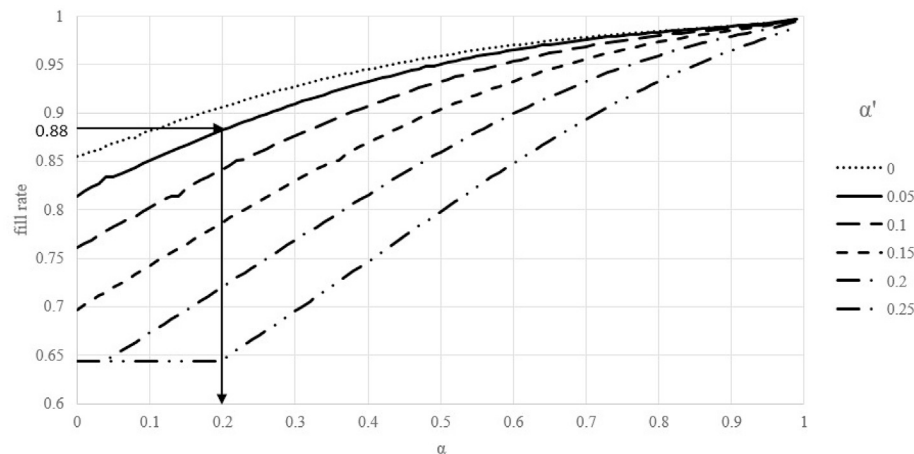


Fig. 7. Service level for different values of α and α' .

The results depicted in Fig. 6 can be used as a decision support tool to set collaboration policies between the two players in a way that mutual interests are served. As mentioned before, when there is no collaboration, the manufacturer can optimize his/her expected total cost and reach a minimum of 18,000. The figure, however, shows that this expected total cost can move to a lower level when a collaboration is in place. This analysis presents all the possible settings for a contractual agreement between the manufacturer and the supplier regarding the values for α and α' . Some combinations of α and α' , however, would be infeasible for one of the players; for instance, the manufacturer can decrease its expected total cost to 15,000 if the supplier agrees to give a discount of 35% and no revenue sharing. Obviously this would not be an option for the supplier since it does not make sense economically.

To reach an agreement and establish a collaboration, the manufacturer considers an expected total cost level as an objective, say 17,500. According to Fig. 6, the supplier should offer a minimum discount of 9% so that the manufacturer considers any collaboration. On the revenue sharing rate curve of $\alpha' = 0$, an expected cost of 17,500 corresponds to a discount rate of $\alpha = 0.09$. The manufacturer would try to reach an agreement with the supplier by offering higher sharing rates (increasing α' on the same cost line). In this example, the manufacturer would ask the supplier for a 20% discount and in return offer 5% of the revenue earned from Q' . If they cannot reach an agreement, perhaps they should aim for a smaller decrease in the cost function and seek possible options. Note that although collaboration may decrease both players' unit marginal profit,

the manufacturer can offer a higher service level and decrease the lost sale. To provide clear managerial insights into how this collaboration would make an impact on the service level offered by the manufacturer, we evaluate the fill rate of the system when α and α' change over the range. The result of this analysis is depicted in Fig. 7. As the figure shows, when there is no collaboration ($\alpha = 0$ and $\alpha' = 0$), the manufacturer can reach a fill rate of 0.85. This fill rate is increased to 0.88 if the above-mentioned agreement is reached ($\alpha = 0.20$ and $\alpha' = 0.05$).

6.2. Price discount contract

In this section, we investigate the effects of price discount through a contractual agreement on production and inventory decisions. The results of this analysis show that when the discount breaking point B , is less than a certain value (for this numerical example this value is 1750¹), the

¹ This value can be obtained through an exhaustive search. After applying a range of values for discount breaking point (B), we find that values above 1750 make order quantities insensitive to discounts offered by the supplier. This is due to the fact that higher discount rates will increase Q' and consequently decrease production quantity, Q . Since we minimize an objective function that contains $E(T)$ (see Equation (23)), lower production quantity would increase the objective function. Therefore, the model avoids the production quantity being less than a certain level. This minimum level can be seen in Fig. 10.

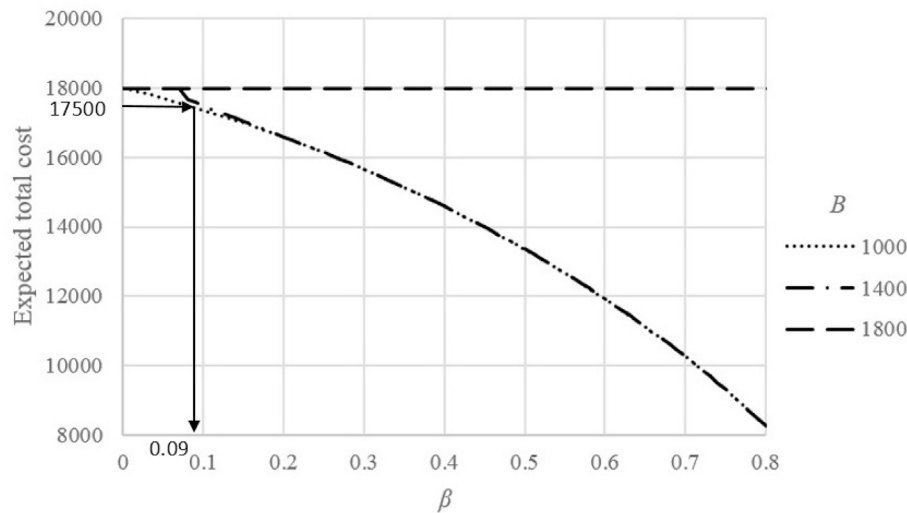


Fig. 8. The effect of discount breaking point on expected total cost.

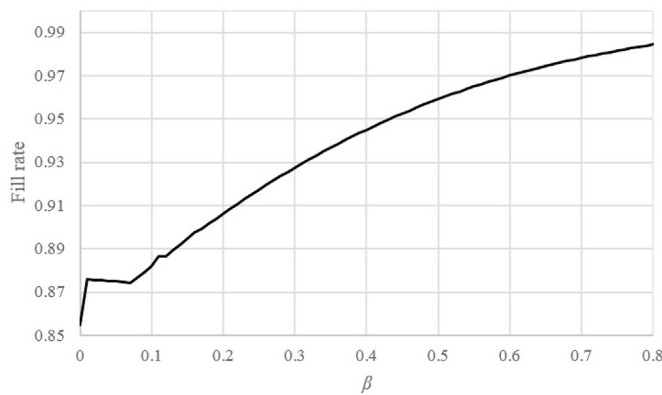


Fig. 9. Fill rate when β changes for $B = 1000$.

expected total cost will decrease in case the discount offered by the supplier increases. Fig. 8 illustrates the effect of discount rates on the expected cost for different values of B . If we set B equal to a value below 1750, say 1000, Fig. 8 shows that higher discount rates result in a lower expected cost. Moreover, the figure shows that when B is greater than 1750, say 1800, the discount option has no effect on the contract, and there will be no changes in the manufacturer's cost no matter what value is assigned to β .

When $B = 1000$, the downturn in the expected cost, presented in Fig. 8, as β increases, is justified when the figure is studied with Figs. 9 and 10. According to Fig. 8, to achieve an expected total cost of 17,500 (instead of 18,000) by the manufacturer, the supplier should offer a discount of 9%. As shown in Fig. 9, this discount results in a higher fill rate for the manufacturer since the manufacturer purchases larger quantities from the supplier (see Fig. 10). From the manufacturer's point of view, this agreement brings about the same gains as the revenue sharing agreement with $\alpha = 0.20$ and $\alpha' = 0.05$. However, the outcomes of these scenarios for the supplier may not be equivalent since we have not taken into account the supplier's objective function. This means the supplier should evaluate the benefits of these scenarios (increase in Q') and their feasibility in accordance with his/her business setting as a whole.

In Fig. 10 ($B = 1000$) both production and order quantity are depicted according to the discount rate the supplier is willing to offer the manufacturer. The figure illustrates that when the supplier offers a higher discount, the manufacturer's production quantity depletes and the order

quantity increases. It should be noted that the system studied in this paper is a production-inventory model and the objective function considers the expected total cost during the *production period*. This means the model uses production as the basis for this business activity and from a mathematical point of view, it does not allow the system to act only as an inventory system (deciding not to produce and fulfilling the demand by only buying quantities from the supplier). This is why high discount rates will not result in zero value for production quantity (see Fig. 10). Therefore, one should consider extreme cases presented in this analysis with more care since some of them would be less practical. For instance, although a higher discount rate offered by the supplier results in a higher fill rate, this increase would be bounded since the demand is constant and the manufacturer can meet the maximum demand of D .

6.3. Supplier's reliability

This study shows that the supplier's reliability can play an important role when making inventory decisions. To conduct the analysis in this section and in Section 6.4, we consider $\alpha = 0.13$, $\alpha' = 0.01$, $B = 1000$ and $\beta = 0.1$. Fig. 11 illustrates how the order quantity that the manufacturer places to the supplier changes under different contracts when the reliability of the supplier varies. In all scenarios, as the figure depicts, the more reliable the supplier, the smaller the order quantity. This is because, in essence, the model is conservative. When the supplier is less reliable, the model suggests the manufacturer purchase larger quantities to hedge against uncertainties and maintain an acceptable fill rate. This balance is obtained through trade-offs made between shortage cost, holding cost, and lost sale cost; for larger lost sale costs, the model strives to avoid shortages by accumulating more inventory. Moreover, Fig. 11 shows that the supplier's reliability has more impact on revenue sharing contract (compare the slope of the line related to the revenue sharing contract to the slope associated with the other collaboration agreements). The results of this study reveal that when there is an agreement in place, the manufacturer can offer a higher fill rate by ordering larger quantities from the supplier. Figs. 11 and 12 give more insights when studied simultaneously: Fig. 12 illustrates that achieving a higher fill rate through a contractual agreement with this specific numerical experiment is possible; and Fig. 11 shows the necessary changes needing to be made to ordering policies, when targeting a specific fill rate.

6.4. Manufacturer's margin

The findings of this research show that the effects of the margins enjoyed by the manufacturer on the order size placed to the supplier

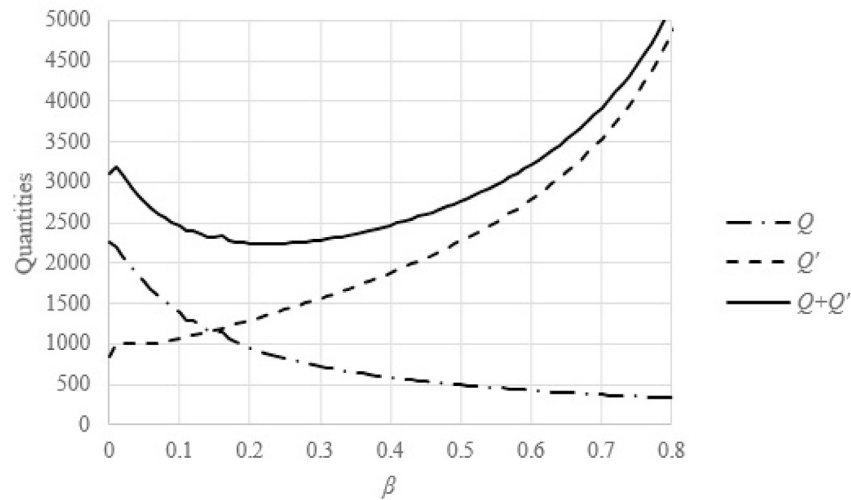


Fig. 10. Production and order quantities when β changes for $B = 1000$.

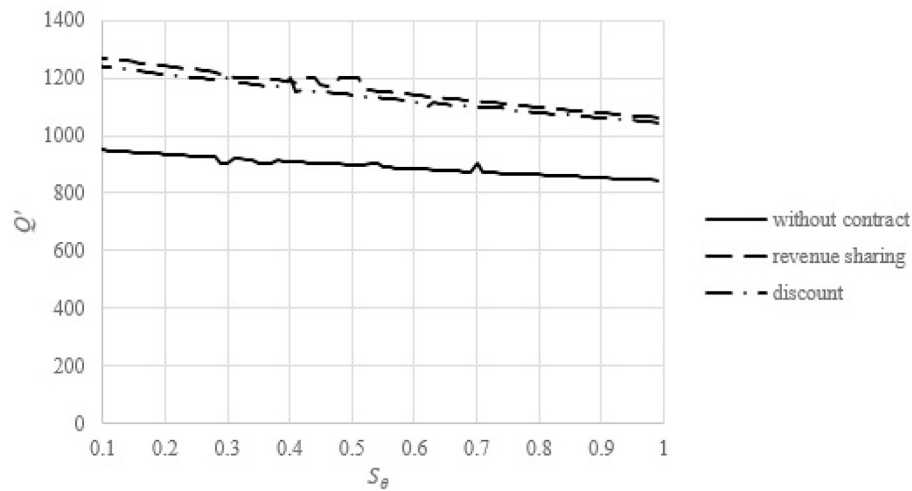


Fig. 11. Q' when S_θ changes in different scenarios.

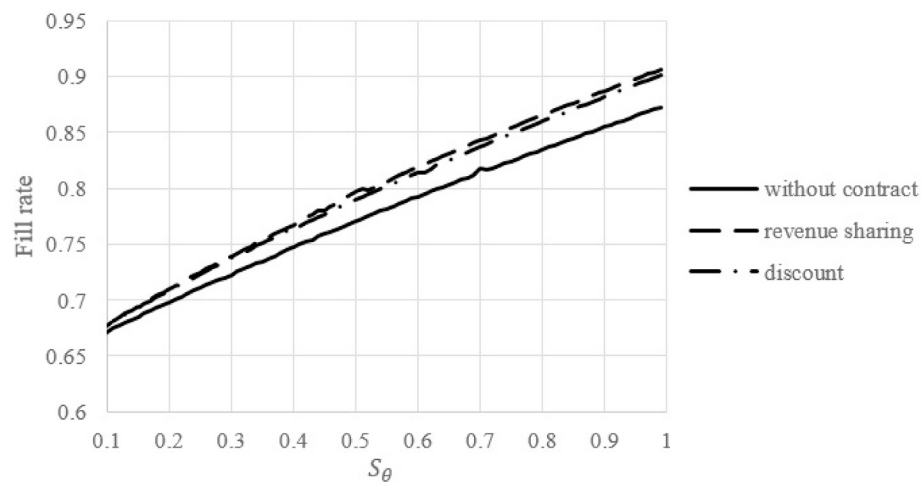


Fig. 12. Fill rate when S_θ changes in different scenarios.

could be significant. To show the influence of the order size placed to the supplier, we define the sales price as $C_p = \gamma C$ and change the value of γ

between 1 and 4 to see how the order quantity changes, see Fig. 13. As the figure illustrates, when γ is small (less than 2.1), the manufacturer

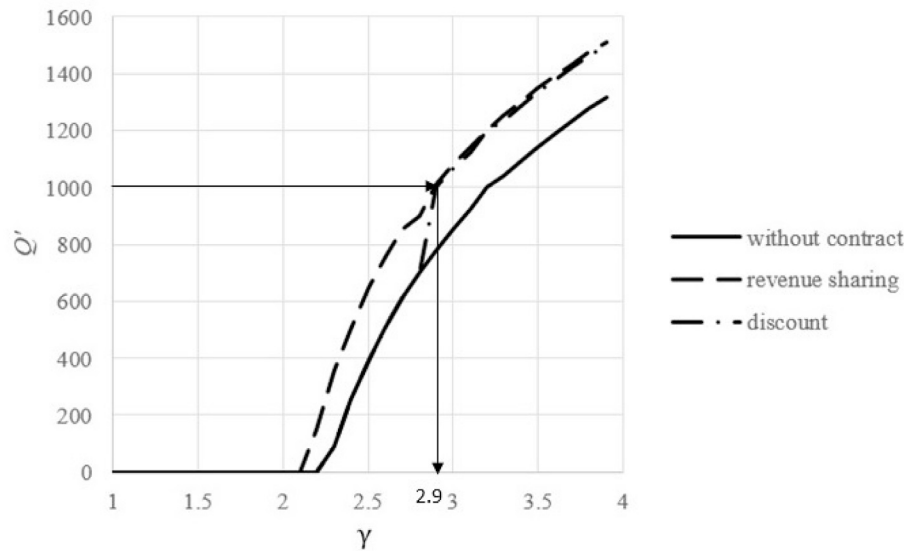


Fig. 13. Q' when selling price changes in different scenarios.

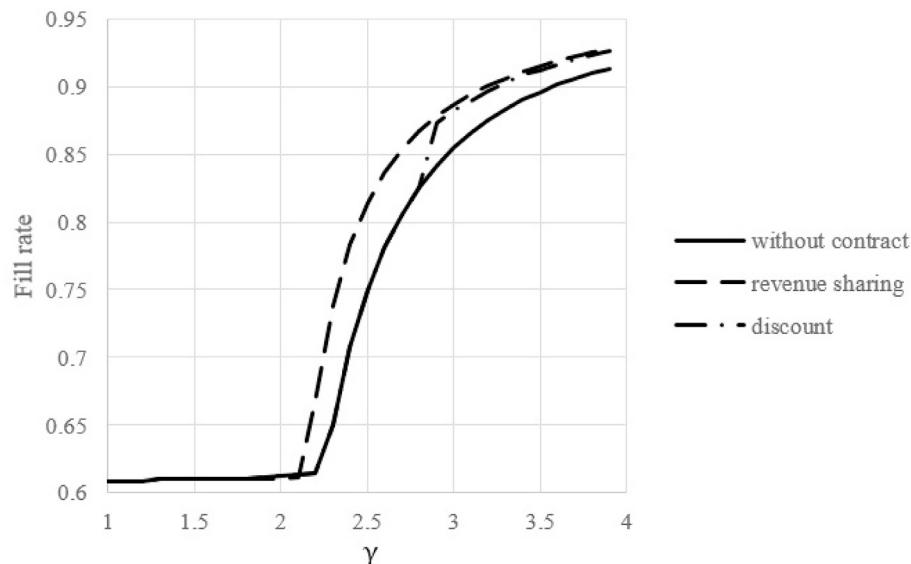


Fig. 14. Fill rate when selling price changes in different scenarios.

tries to meet the demand by relying on his/her production system without considering any collaboration. When dealing with items with higher margins, however, Fig. 14 shows that the manufacturer aims for a higher fill rate and seeks collaboration, preferably in the form of a revenue sharing contract with the supplier. This is intuitive because when higher margins are involved, losing a unit of demand results in a higher cost to the manufacturer. According to Fig. 13 when $\gamma = 2.1$ revenue sharing is the dominant option compared to cases without a contract or with a discount contract. For $\gamma > 2.9$ a contractual agreement with price discount becomes beneficial. This is due to the fact that when $\gamma = 2.9$, Q' reaches 1000 which is the breakpoint for the discount contract.

6.5. Comparison with the safety stock policy

Our numerical experiments show that the parameter with the most influence on the purchasing scenarios and safety stock policy is the holding cost of the manufacturer's items. To do such an experiment, we increase the unit production cost (which linearly increases the holding cost) over a range from C to $2C$. The model is then solved for all the

scenarios (without a contract, revenue sharing with $\alpha = 0.13$ and $\alpha' = 0.01$ and quantity discount with $B = 1000$ and $\beta = 0.1$) to obtain the corresponding expected total cost and fill rate. The results of this numerical analysis are illustrated in Figs. 15 and 16. In these figures, the horizontal axis (η) represents the coefficient for the unit production cost in the original example.

As Fig. 15 shows, for the data set discussed in this paper, the safety stock is still a better option than purchasing without a contract as long as there is an increase of up to 10% in the unit production cost; however, higher values for unit production cost make the safety stock policy less economical. The figure depicts a better performance for the manufacturer when there is an agreement in place.

The safety stock policy offers a higher service level than other scenarios for the original dataset. The superiority of the safety stock policy remains unchanged if there is an increase of up to 30% in the unit production cost (see Fig. 16). Further increase in the unit production cost, however, does not result in higher fill rate for the safety stock scenario since the model makes trade-offs between the holding costs and the shortage costs. In this situation, if an agreement, such as the purchasing

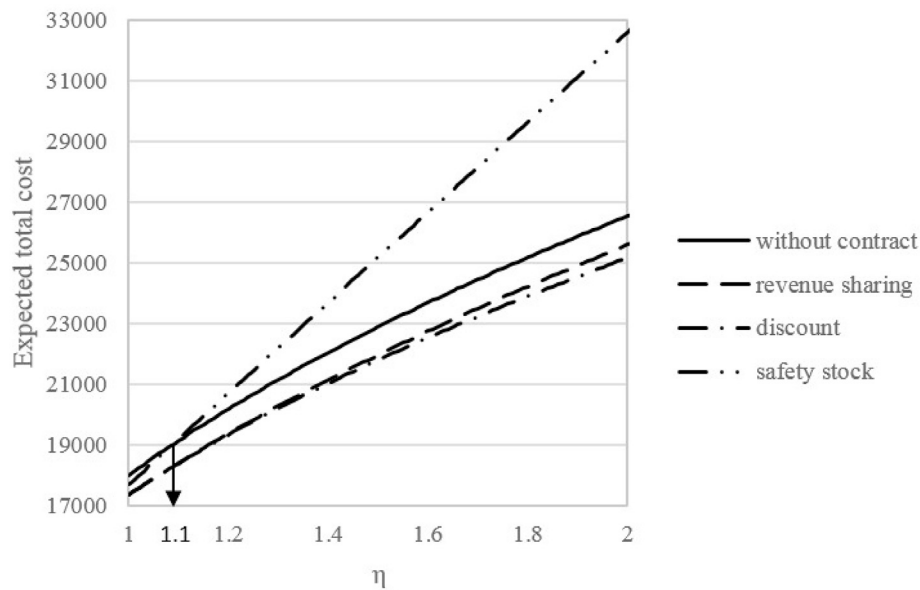


Fig. 15. Expected total cost when production cost changes in different scenarios.

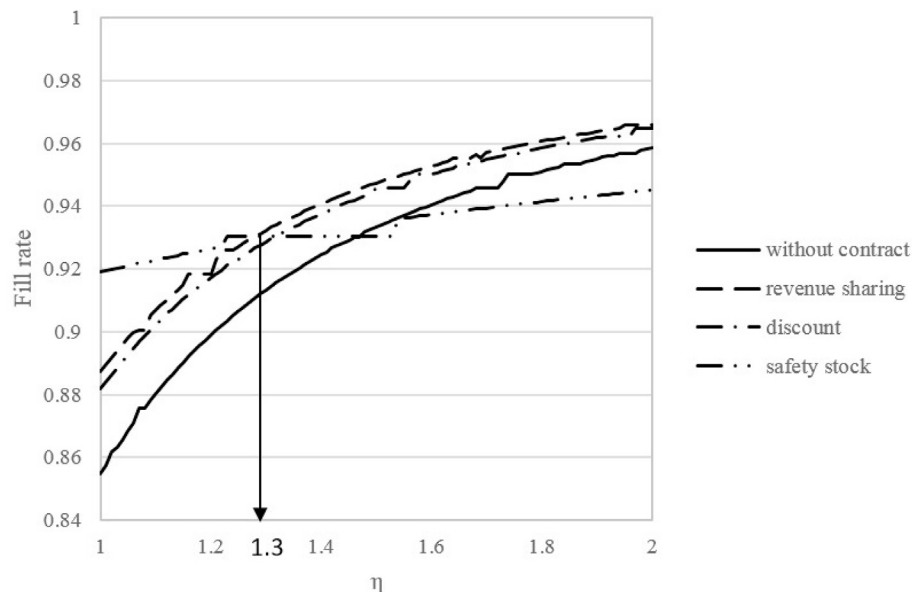


Fig. 16. Fill rate when production cost changes in different scenarios.

scenario, has been reached, the manufacturer can offer a higher level of service.

7. Conclusion

In this paper, we have developed and analyzed a production system that is subject to failure. We investigate how the manufacturer can benefit from the existence of an external supplier in the event of a breakdown in the production process. To do so, we evaluate three purchasing scenarios: buying items from a supplier with no contractual agreement, purchasing from a supplier through a price discount contract, and ordering quantities based on a revenue-sharing contract with a supplier. In contrast to studies by others which focus on preventive maintenance, repair operations, and holding safety stock, our results show that the level of service and expected total cost can be improved if the manufacturer collaborates with other businesses.

In this paper, we show how the unit production cost, and hence the

unit holding cost, play an essential role in determining when it is better to collaborate with a supplier rather than rely on keeping safety stock. The study also shows that for less expensive items, storing safety stock would be a better option since it offers a higher fill rate. This, however, would not be the case for more expensive items. The analysis conducted in this paper indicates that collaborating with a supplier, in the event of a system breakdown, could significantly decrease the expected total cost while offering a reasonably high fill rate.

Implementing such agreements is challenging since players need to accept some compromise on their short-term gains, but worthwhile, as this study has shown. Both the manufacturer and the supplier stand to gain a higher share of the market by increasing the fill rate. This collaboration will only be effective if there is a predetermined process for quantifying and distributing the gains amongst the players.

The model introduced in this paper could be improved upon by including information from the supplier's perspective. We suggest that this model be applied to a real dataset which includes data about the

supplier's cost function. This would facilitate the use of contracts specifically developed for a given item and the relevant market. It could be further extended by considering the option that the manufacturer both purchases from an available supplier and holds safety stock. This might require less safety inventory to be held and protect the company against stock-outs.

Another avenue for further research is to consider the manufacturer and supplier's cash constraints and how the weaker player might be strengthened in this supply chain. For example, a cash-flow based objective function could be developed, with the two players agreeing on the timing of the payments and a clear cut process as to how the benefits would be shared amongst them.

Appendix A. Detailed analysis for $E(T)$ and $E(TC)$ in Scenario 1

In this appendix we provide the details of the analysis performed in Section 4.1 for obtaining $E(T)$ and $E(TC)$.

$$\begin{aligned}
 E(T) &= \int_0^{t_p} \lambda e^{-\lambda t_w} \int_0^\infty (T_2 + T_3 + T_4 + T_5) \mu e^{-\mu t_r} dt_r dt_w + \int_{t_p}^\infty T_1 \lambda e^{-\lambda t_w} dt_w \\
 &= \int_0^{t_p} \lambda e^{-\lambda t_w} \int_0^{t_o} T_2 \mu e^{-\mu t_r} dt_r dt_w + (1 - S_\theta) \int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o}^\infty T_3 \mu e^{-\mu t_r} dt_r dt_w \\
 &+ S_\theta \left(\int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o}^{t_o+t_m} T_4 \mu e^{-\mu t_r} dt_r dt_w + \int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o+t_m}^\infty T_5 \mu e^{-\mu t_r} dt_r dt_w \right) + \int_{t_p}^\infty T_1 \lambda e^{-\lambda t_w} dt_w \\
 &+ \int_0^{t_p} \frac{P t_w}{D} \lambda e^{-\lambda t_w} \int_0^{t_o} \mu e^{-\mu t_r} dt_r dt_w + (1 - S_\theta) \int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o}^\infty (t_w + t_r) \mu e^{-\mu t_r} dt_r dt_w \\
 &+ S_\theta \left(\int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o}^{t_o+t_m} \left(\frac{P t_w}{D} + t_m \right) \mu e^{-\mu t_r} dt_r dt_w + \int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o+t_m}^\infty (t_w + t_r) \mu e^{-\mu t_r} dt_r dt_w \right) \\
 &+ \int_{t_p}^\infty \frac{Q}{D} \lambda e^{-\lambda t_w} dt_w
 \end{aligned} \tag{24}$$

$$\begin{aligned}
 &= \left(\frac{\lambda}{\mu} (1 - S_\theta) + S_\theta \lambda t_m + \frac{S_\theta \lambda e^{-\mu t_m}}{\mu} \right) \left(\frac{1 - e^{-(\lambda + \frac{\mu(P-D)}{D}) t_p}}{\lambda + \frac{\mu(P-D)}{D}} \right) + \frac{P}{\lambda D} (1 - e^{-\lambda t_p}) \\
 &= \left(\frac{\lambda}{\mu} (1 - S_\theta) + S_\theta \lambda \frac{Q'}{D} + \frac{S_\theta \lambda e^{-\mu \frac{Q'}{D}}}{\mu} \right) \left(\frac{1 - e^{-(\lambda + \frac{\mu(P-D)}{D}) \frac{Q'}{D}}}{\lambda + \frac{\mu(P-D)}{D}} \right) + \frac{P}{\lambda D} (1 - e^{-\lambda \frac{Q'}{D}}) \\
 E(TC) &= \int_0^{t_p} \lambda e^{-\lambda t_w} \int_0^\infty (TC_2 + TC_3 + TC_4 + TC_5) \mu e^{-\mu t_r} dt_r dt_w + \int_{t_p}^\infty TC_1 \lambda e^{-\lambda t_w} dt_w \\
 &= \int_0^{t_p} \lambda e^{-\lambda t_w} \int_0^{t_o} TC_2 \mu e^{-\mu t_r} dt_r dt_w + (1 - S_\theta) \int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o}^\infty TC_3 \mu e^{-\mu t_r} dt_r dt_w \\
 &+ S_\theta \left(\int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o}^{t_o+t_m} TC_4 \mu e^{-\mu t_r} dt_r dt_w + \int_0^{t_p} \lambda e^{-\lambda t_w} \int_{t_o+t_m}^\infty TC_5 \mu e^{-\mu t_r} dt_r dt_w \right) \\
 &+ \int_{t_p}^\infty TC_1 \lambda e^{-\lambda t_w} dt_w \\
 &= A + \left(\frac{\lambda \left(1 - e^{-(\lambda + \frac{\mu(P-D)}{D}) t_p} \right)}{\lambda + \frac{\mu(P-D)}{D}} \right) \left(S_\theta \left(A' + \frac{h' D t_m^2}{2} + C' D t_m \right) + \left(\frac{\pi D}{\mu} (1 - S_\theta + S_\theta e^{-\mu t_m}) \right) \right) \\
 &\quad + \left(\frac{C_r}{\mu} + \frac{CP}{\lambda} \right) (1 - e^{-\lambda t_p}) + \frac{h(P-D)}{D} \left(\frac{P(1 - e^{-\lambda t_p})}{\lambda^2} - \frac{P t_p e^{-\lambda t_p}}{\lambda} \right) \\
 &= A + \left(\frac{\lambda \left(1 - e^{-(\lambda + \frac{\mu(P-D)}{D}) \frac{Q'}{D}} \right)}{\lambda + \frac{\mu(P-D)}{D}} \right) \left(S_\theta \left(A' + \frac{h' Q'^2}{2} + C' Q' \right) + \left(\frac{\pi D}{\mu} (1 - S_\theta + S_\theta e^{-\mu \frac{Q'}{D}}) \right) \right) \\
 &\quad + \left(\frac{C_r}{\mu} + \frac{CP}{\lambda} \right) (1 - e^{-\lambda \frac{Q'}{D}}) + \frac{h(P-D)}{D} \left(\frac{P(1 - e^{-\lambda \frac{Q'}{D}})}{\lambda^2} - \frac{Q e^{-\lambda \frac{Q'}{D}}}{\lambda} \right)
 \end{aligned} \tag{25}$$

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